IIT Mandi
Proposal for a New Course

Course number : MA-528
Course Name : Measure Theory and Integration
Credit Distribution : (3-1-0-4)
Intended for : M.Sc./M.S./PhD/B.Tech
Prerequisite : MA-511 (Real Analysis)
Mutual Exclusion : NA

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1. Preamble:

This one semester course is designed to provide exposure of the concept of measure theory and to teach the students how integration can be developed in terms of measure. The course starts with a review of Riemann integral, which is defined on a closed and bounded subset of $\mathbb{R}$. The main idea of Lebesgue integration is to generalize the concept of Riemann integration and to formulate an abstract notion of integration. We will define the essential L^p spaces which are frequently used in Harmonic analysis and Partial differential equation. In this course we will cover all the basic topics: Properties of Lebesgue Integral, Lebesgue Dominated Convergence Theorem, Fundamental Theorem of Calculus, Radon-Nikodym Theorem, Fubini's Theorem and so on.
Apart from measure theory has central importance in pure mathematics, it has also uses in different branches of applied mathematics as well in Fourier analysis, Probability theory etc.

2. Course Modules with quantitative lecture hours:

Module 1: Review of Riemann integral, Algebra of subsets of a non-empty set, Measure on an arbitrary sigma-algebra, Continuity property of measure, The induced outer measure, Measurable sets, Borel Sigma algebra, Monotone class, Completion of a measure space , The Lebesgue measure on $\mathbb{R}$, Properties of Lebesgue measure, Non measurable subsets of $\mathbb{R}$.

(14 hours)

Module 2: Simple measurable functions, Integral of non-negative measurable functions, Monotone convergence theorem, Fatou's Lemma, Dominated convergence theorem, Relation between Riemann , Improper and Lebesgue integrals , Riesz-Fischer theorem (L_1[a,b] is a complete metric space) , R[a,b] is dense L_1[a,b], Lusin's theorem, Lp- spaces, Convergence of measurable functions (almost everywhere , in measure, in mean).

(16 Hours)

Module 3: Absolutely continuous functions, Differentiability of monotone functions (Only statement of Lebesgue-Young theorem), Fundamental theorem of calculus for Lebesgue integrable functions, Radon-Nikodym theorem, Product measure, Fubini's theorem, Signed measure, Riesz representation theorem (Without proof) .

(12 Hours)

3. Text books:

1. I. K. Rana, An introduction to Measure and Integration, Second Edition, Narosa, 2005.
2. G. de Barra, Measure and Integration, Wiley Eastern, 1981.

4. References:

1. W. Rudin, Real and Complex Analysis, Third edition, McGraw-Hill, International Editions, 1987.
2. H. L. Royden, Real Analysis, Third edition, Prentice-Hall of India, 1985.
3. G. B. Folland, "Real Analysis", Wiley-Interscience Publication, John Wiley & Sons, 1999.
4. M. Thamban Nair, Measure and Integration, A first course, CRC Press, 2020.

5. Similarity with the existing courses:
(Similarity content is declared as per the number of lecture hours on similar topics)

S. No.		Course code	Similarity content	Approx.% of content
1.	Riemann integration	MA-511	2 hours	~4.76%
2.	Measurable space	MA-524	1 hour	~2.38%